T-odd correlations in $\pi \to e\nu_e\gamma$ and $\pi \to \mu\nu_\mu\gamma$ decays

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The transverse lepton polarization asymmetry in $\pi_{l2\gamma}$ decays may probe *T*-violating interactions beyond the Standard Model. Dalitz plot distributions of the expected effects are presented and compared to the contribution from the Standard Model final-state interactions. We give an example of a phenomenologically viable model, where a considerable contribution to the transverse lepton polarization asymmetry arises.

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I. INTRODUCTION

T violation beyond the Standard Model (SM) is usually searched for in decays forbidden by time reversal symmetry. Another way to probe *T* violation is the measurement of *T*-odd observables in allowed decays of mesons. A well known example is $K^0 \to \pi^+\pi^-e^+e^-$ decay, where the *T*-odd correlation is experimentally observed [1] and coincides with the theoretical prediction of the Standard Model [2]. Other widely considered *T*-odd observables are transverse muon polarizations (P_T) in $K \to \pi\mu\nu$ and $K \to \mu\nu\gamma$ decays. There is no tree level SM contribution to P_T in these decays, so they are of a special interest for the search for new physics. Unfortunately, P_T is not exactly zero in these decays even in *T*-invariant theory—electromagnetic loop corrections contribute to P_T and should be considered as a background. There is no experimental evidence for nonzero P_T in these processes at the present time [3,4], but the sensitivity of the experiments has not yet reached the level of SM loop contributions.

In this paper we study the decays $\pi \to e\nu\gamma$ and $\pi \to \mu\nu\gamma$. Within the Standard Model, *T* violation in these processes does not appear at tree level, but interactions contributing to it emerge in various extensions of the SM. We shall demonstrate that $\pi_{l2\gamma}$ decays are attractive probes of new physics beyond the Standard Model. Depending on the model, $\pi_{l2\gamma}$ decays may be even more attractive than the usually considered $K_{l2\gamma}$ decays.

Although $\pi_{e2\gamma}$ decay has a very small branching ratio (it is of order 10^{-7}), we find that the distribution of the transverse electron polarization over the Dalitz plot significantly overlaps with the distribution of the differential branching ratio, as opposed to the situation with $K_{\mu2\gamma}$ decay. Moreover, the contribution of FSI (final-state interactions related to SM one-loop diagrams) to the observable asymmetry, being at the level of 10^{-3}, becomes even smaller in that region of the Dalitz plot where the contribution from the new effective *T*-violating interaction is maximal. Thus, $\pi_{e2\gamma}$ decay is potentially quite an interesting probe of *T* violation, although it is worth noting that the experimental measurement of electron or positron polarization is quite complicated.

The $\pi_{\mu2\gamma}$ decay has a much higher branching ratio than $\pi_{e2\gamma}$ decay, and experimental measurement of muon polarization is simpler than that of the electron. Unlike in $\pi_{e2\gamma}$ decay, however, FSI contributions and contributions from *T*-violating interactions to the transverse muon polarization are of similar shape, as we show in this paper. This means that only those new physics effects may be detected which are stronger than FSI, but if this is the case then detecting *T* violation in $\pi_{\mu2\gamma}$ needs much smaller pion statistics than that required in the case of $\pi_{e2\gamma}$.

To demonstrate that pion decays may be relevant processes where the signal of new physics may be searched for, we present a simple model of heavy pseudoscalar particle exchange leading in the low energy limit to a *T*-violating four-fermion interaction. We find the constraints on the parameters of this model coming from various other experiments and describe regions of the parameter space which result in large *T*-violating effects in $\pi_{l2\gamma}$ decays. Depending on the parameters of the model, an experiment measuring transverse lepton polarization with pion statistics of 10^5–10^{10} pions for $\pi_{\mu2\gamma}$ decay and 10^8–10^{13} pions for $\pi_{e2\gamma}$ decay is needed to detect the *T*-violating effects (taking into account statistical uncertainty only and assuming ideal experimental efficiencies).

The paper is organized as follows. In Sec. II we introduce a generic effective four-fermion interaction giving rise to *T*-odd correlation of lepton transverse polarization and calculate the distributions of this polarization and of the differential branching ratio of $\pi_{l2\gamma}$ decay over the Dalitz plot. In Sec. II A we estimate the contribution of the final-state interactions to the observable asymmetry. Section II B is devoted to the constraint on the effective Lagrangian coming from the measurement of $\pi \to l\nu_l$ decays. Generically, this constraint is quite strong, but it becomes much less restrictive if there is a hierarchy of the constants in the Lagrangian responsible for the observable *T*-violating effects. An example of a high energy model of *T* violation is presented in Sec. III. The constraints on the parameters of this model emerging from the measurements of muon lifetime, from the study of the parameters of kaon mixing, and from the experimental limit on the neutron dipole moment are considered in Secs. III A, III B, and III C, respectively. It is shown that for generic values of the model parameters the measurement of the CP-violating parameter ϵ forbids new observable *T*-violating effects in $\pi_{l2\gamma}$ decays, but if the parameters ex-

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hibit a special pattern, the T-violating effects in $\pi_{l2\gamma}$ may be large.

II. T-VIOLATING EFFECT IN $\pi^+_{l2\gamma}$ DECAY

Let us consider the simplest effective four-fermion interaction[1]

$$\mathcal{L}_{\rm eff}=G^l_{\rm P}\overline{d}\gamma_5 u\cdot\overline{\nu_l}(1+\gamma_5)l+\text{H.c.} \tag{1}$$

that may be responsible for T-violating effects in pion physics beyond the Standard Model. Indeed, the imaginary part of the constant $G^l_{\rm P}$ contributes to transverse lepton polarization in decays $\pi^+_{l2\gamma}$.

To calculate this polarization let us write the amplitude of the decay $\pi^+_{l2\gamma}$ in terms of inner bremsstrahlung (IB) and structure-dependent (SD) contributions [5–7]:

$$M=M_{\rm IB}+M_{\rm SD},$$

with

$$M_{\rm IB}=ie\frac{G_F}{\sqrt{2}}\cos\theta_c f_\pi m_l\epsilon^*_\alpha K^\alpha,$$

$$M_{\rm SD}=-ie\frac{G_F}{\sqrt{2}}\cos\theta_c\epsilon^*_\mu L_\nu H^{\mu\nu}, \tag{2}$$

where

$$K^\alpha=\overline{u}(k)(1+\gamma_5)\left(\frac{p^\alpha}{pq}-\frac{2p_l^\alpha+\hat{q}\gamma^\alpha}{2p_lq}\right)v(p_l,s_l),$$

$$L_\nu=\overline{u}(k)\gamma_\nu(1-\gamma_5)v(p_l,s_l),$$

$$H^{\mu\nu}=\frac{F_A}{m_\pi}(-g^{\mu\nu}pq+p^\mu q^\nu)+i\frac{F_V}{m_\pi}\varepsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta, \tag{3}$$

with the convention for the Levi-Cività tensor $\varepsilon^{0123}=-1$. Here ϵ_α is the photon polarization vector, p, k, p_l, and q are the four-momenta of π^+, ν_l, l^+, and γ, respectively, s_l is the polarization vector of the charged lepton, F_V and F_A are vector and axial-vector form factors of pion radiative decay; the effect coming from the Lagrangian (1) may be absorbed in the f_π form factor

$$f_\pi=f^0_\pi(1+\Delta^l_P),\quad f^0_\pi\approx 130.7\ \text{MeV}, \tag{4}$$

with

$$\Delta^l_P=\frac{\sqrt{2}G^l_P}{G_F\cos\theta_c}\cdot\frac{B_0}{m_l},$$

$$B_0=-\frac{2}{(f^0_\pi)^2}\langle 0|\overline{q}q|0\rangle=\frac{m^2_\pi}{m_u+m_d}\approx 2\ \text{GeV}.$$

We write the components of s_l in terms of $\vec{\xi}$, a unit vector along the lepton spin in its rest frame, as follows:

$$s_0=\frac{\vec{\xi}\vec{p}_l}{m_l},\quad \vec{s}=\vec{\xi}+\frac{s_0}{E_l+m_l}\vec{p}_l.$$

In the rest frame of π^+, the partial decay width into the state with lepton spin $\vec{\xi}$ is found to be

$$d\Gamma(\vec{\xi})=\frac{1}{2m_\pi}|M|^2(2\pi)^4\delta(p-p_e-k-q)\times\frac{d\vec{q}}{2E_q(2\pi)^3}\frac{d\vec{p}_l}{2E_l(2\pi)^3}\frac{d\vec{k}}{2E_\nu(2\pi)^3},$$

with

$$|M|^2=\rho_0(x,y)[1+(P_L\vec{e}_L+P_N\vec{e}_N+P_T\vec{e}_T)\cdot\vec{\xi}]$$

where $\vec{e}_i$ $(i=L,N,T)$ are the unit vectors along the longitudinal (P_L), normal (P_N), and transverse (P_T) components of the lepton polarization, defined by

$$\vec{e}_L=\frac{\vec{p}_l}{|\vec{p}_l|},\quad \vec{e}_N=\frac{\vec{p}_l\times(\vec{q}\times\vec{p}_l)}{|\vec{p}_l\times(\vec{q}\times\vec{p}_l)|},\quad \vec{e}_T=\frac{\vec{q}\times\vec{p}_l}{|\vec{q}\times\vec{p}_l|}, \tag{5}$$

respectively,

$$\rho_0(x,y)=e^2\frac{G_F^2}{2}\cos^2\theta_c(1-\lambda)\left\{\frac{4m_l^2|f_\pi|^2}{\lambda x^2}\left[x^2+2(1-r_l)\left(1-x-\frac{r_l}{\lambda}\right)\right]+m_\pi^4x^2\left[\left|F_V+F_A\right|^2\frac{\lambda^2}{1-\lambda}\left(1-x-\frac{r_l}{\lambda}\right)\right.\right.$$
$$\left.\left.+\left|F_V-F_A\right|^2(y-\lambda)\right]-4m_\pi m_l^2\left[\text{Re}[f_\pi(F_V+F_A)^*]\left(1-x-\frac{r_l}{\lambda}\right)-\text{Re}[f_\pi(F_V-F_A)^*]\frac{1-y+\lambda}{\lambda}\right]\right\},$$

[1]Additional scalar, vector, or axial type interactions do not contribute to lepton transverse polarization [5].

with $\lambda=(x+y-1-r_l)/x$, $r_l=m_l^2/M_\pi^2$, and $x=2pq/p^2=2E_\gamma/m_\pi$ and $y=2pp_l/p^2=2E_l/m_\pi$ are the normalized energies of the photon and charged lepton, respectively. In terms of these variables the differential decay width reads

$$d\Gamma(\vec{\xi})=\frac{m_\pi}{32(2\pi)^3}|M(x,y,\vec{\xi})|^2 dx\, dy$$

$$=\frac{m_\pi}{32(2\pi)^3}|M(x,\lambda,\vec{\xi})|^2 x dx d\lambda. \tag{6}$$

For the transverse lepton polarization asymmetry

$$P_T(x,y)=\frac{d\Gamma(\vec{e}_T)-d\Gamma(-\vec{e}_T)}{d\Gamma(\vec{e}_T)+d\Gamma(-\vec{e}_T)}, \tag{7}$$

we find

$$P_T(x,y)=\frac{\rho_T(x,y)}{\rho_0(x,y)},$$

with

$$\rho_T(x,y)=-2e^2G_F^2\cos^2\theta_c m_\pi^2 m_l \frac{1-\lambda}{\lambda}\sqrt{\lambda y-\lambda^2-r_l}$$

$$\times\left\{\mathrm{Im}[f_\pi(F_V+F_A)^*]\frac{\lambda}{1-\lambda}\left(1-x-\frac{r_l}{\lambda}\right)\right.$$

$$\left.+\mathrm{Im}[f_\pi(F_V-F_A)^*]\right\}.$$

The asymmetry P_T is odd under time reversal (the sign of $\vec{\xi}\vec{e}_T$ obviously changes under time reversal), and P_N and P_L, defined analogously to Eq. (7), are even under time reversal. Moreover, one can show that interaction (1) does not contribute to P_N and P_L [5]. One observes that P_T is proportional to the imaginary part of Δ_P, so it is convenient to rewrite Eq. (7) in the following form:

$$P_T(x,y)=[\sigma_V(x,y)-\sigma_A(x,y)]\cdot\mathrm{Im}[\Delta_P^l], \tag{8}$$

where

$$\sigma_V(x,y)=2e^2G_F^2\cos^2\theta_c m_\pi^2 m_l f_\pi^0 F_V$$

$$\times\frac{\sqrt{\lambda y-\lambda^2-r_l}}{\rho_0(x,y)}\left[\frac{\lambda-1}{\lambda}-\left(1-x-\frac{r_l}{\lambda}\right)\right],$$

$$\sigma_A(x,y)=2e^2G_F^2\cos^2\theta_c m_\pi^2 m_l f_\pi^0 F_A$$

$$\times\frac{\sqrt{\lambda y-\lambda^2-r_l}}{\rho_0(x,y)}\left[\frac{\lambda-1}{\lambda}+\left(1-x-\frac{r_l}{\lambda}\right)\right].$$

Taking $F_V=-0.0259$ [conserved vector current (CVC) prediction] and $F_A=-0.0116$ [8,9] we present in Fig. 1 the contour plot of $[\sigma_V-\sigma_A]$ as a function of x and y for $\pi_{e2\gamma}$ and $\pi_{\mu 2\gamma}$ decays.

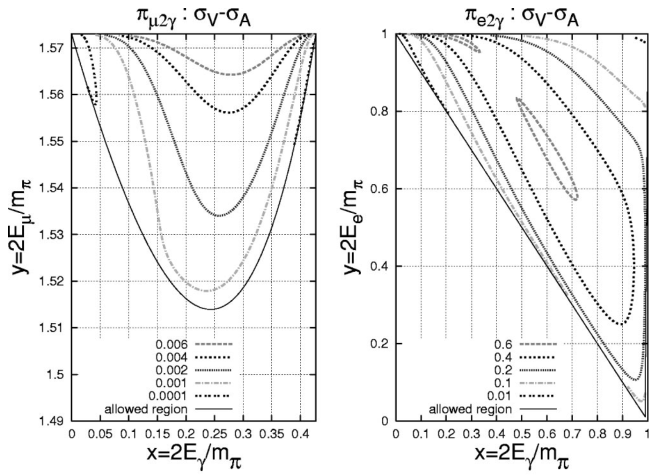

FIG. 1. The contour plots for the function $[\sigma_V-\sigma_A]$ for $\pi_{\mu 2\gamma}$ and $\pi_{e2\gamma}$ decays, which describe the P_T distribution over the Dalitz plot [see Eq. (8)].

As one can see, in a large region of kinematic variables, $[\sigma_V-\sigma_A]$ is about 0.5 for the $\pi_{e2\gamma}$ decay. This means that the transverse electron polarization P_T for this process, Eq. (8), is of the same order as $\mathrm{Im}[\Delta_P^e]\simeq 5\times 10^3\cdot\mathrm{Im}[G_P^e/G_F]$. It is worth noting that the region of the Dalitz plot where a large T-violating effect might be observed significantly overlaps with the region where the partial decay width $\Gamma(\pi_{e2\gamma})$ is saturated (cf. Figs. 1 and 2). This is in contrast to the situation with T violation in $K_{\mu 2\gamma}$ decay (see, e.g., Ref. [5]), where the analogous overlap is small, so the differential branching ratio in the relevant region is smaller than on average.

The situation with $\pi_{\mu 2\gamma}$ decay is less attractive: one finds that $\mathrm{Im}[\Delta_P^\mu]\simeq 25\mathrm{Im}[G_P^\mu/G_F]$ and $[\sigma_V-\sigma_A]$ is of the order of 10^{-3}–10^{-4} over the Dalitz plot. Moreover, the decay rate is saturated in the region of small x (low energy photons), where the T-violating effect is small. On the other hand, the branching ratio is larger by three orders of magnitude than that of $\pi_{e2\gamma}$ decay.

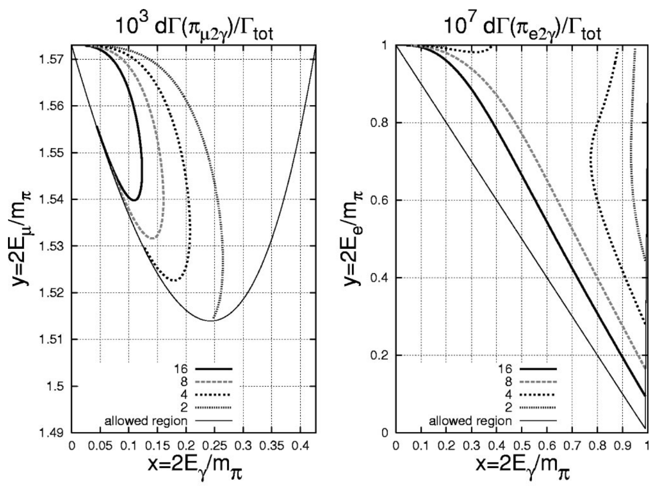

FIG. 2. The distribution of differential branching ratios of $\pi_{\mu 2\gamma}$ and $\pi_{e2\gamma}$ decays (multiplied by 10^3 and 10^7, respectively) over the Dalitz plot.

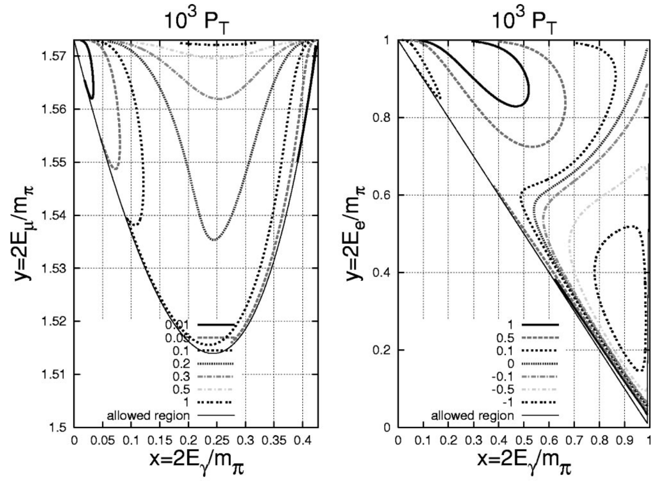

FIG. 3. Transverse lepton polarization due to FSI in $\pi_{\mu 2\gamma}$ and $\pi_{e2\gamma}$ decays.

A. Final-state interactions

Now let us estimate the SM contribution to the T-violating observable P_T. This contribution arises due to final-state interactions—one-loop diagrams with virtual photons. These diagrams are similar to the diagrams leading to the FSI contribution in $K\to\mu\nu_\mu\gamma$ decay. The latter contribution was calculated in Ref. [10]. Thus FSI in $\pi_{l2\gamma}$ may be estimated by making use of the corresponding replacements ($m_K\to m_\pi$, etc.) in the formulas of Ref. [10]. The result is presented in Fig. 3.

For the $\pi_{e2\gamma}$ decay, the (x,y) distributions of the FSI contribution and the contribution from the four-fermionic interaction (1) differ in shape. Specifically, part of the region with maximal P_T from the four-fermion interaction (1) corresponds to the region of small P_T from FSI. This implies that, if measured, the P_T distribution could probe the T-violating interaction (1) with an accuracy higher than $\mathrm{Im}[\Delta_P^e]\sim 10^{-3}$ ($\mathrm{Im}[G_P^e/G_F]\sim 2\times 10^{-7}$). Again, this is not the case for $K_{\mu 2\gamma}$ decay (see Ref. [10]).

For the $\pi_{\mu 2\gamma}$ decay the situation is less promising. The contributions from the Lagrangian (1) and from FSI are distributed over the Dalitz plot similarly. So one may hope to probe T-violating interaction only at the level $\mathrm{Im}[\Delta_P^\mu]\gtrsim 10^{-1}$ ($\mathrm{Im}[G_P^\mu/G_F]\sim 3\times 10^{-3}$).

B. Constraint from $\pi\to l\nu$ decays

The interaction term (1) not only gives rise to T violation in $\pi\to l\nu_l\gamma$ decays but also contributes to the rate of $\pi\to l\nu_l$ decays. Since the ratio of leptonic decays of the pion has been accurately measured [8,11–13],

$$R=\frac{\Gamma(\pi\to e\nu)+\Gamma(\pi\to e\nu\gamma)}{\Gamma(\pi\to\mu\nu)+\Gamma(\pi\to\mu\nu\gamma)}=(1.230\pm 0.004)\times 10^{-4}, \quad (9)$$

the coupling constants G_P^e and G_P^μ are strongly constrained. Indeed, the standard $(V-A)$ theory of electroweak interactions predicts

$$R_0=\frac{m_e^2}{m_\mu^2}\frac{(m_\pi^2-m_e^2)^2}{(m_\pi^2-m_\mu^2)^2}(1+\delta)=1.233\times 10^{-4},$$

where δ is the radiative correction [14–17]. The four-fermion interaction (1) changes the prediction:

$$R=R_0\frac{|1+\Delta_P^e|^2}{|1+\Delta_P^\mu|^2}=R_0\frac{1+2\mathrm{Re}[\Delta_P^e]+(\mathrm{Re}[\Delta_P^e])^2+(\mathrm{Im}[\Delta_P^e])^2}{1+2\mathrm{Re}[\Delta_P^\mu]+(\mathrm{Re}[\Delta_P^\mu])^2+(\mathrm{Im}[\Delta_P^\mu])^2}.$$

Thus to the second order in Δ_P one obtains the constraint

$$-2.9\times 10^{-3}<f(\Delta_P^e,\Delta_P^\mu)<0.4\times 10^{-3}, \quad (10)$$

$$f(\Delta_P^e,\Delta_P^\mu)=\mathrm{Re}[\Delta_P^e-\Delta_P^\mu]+\frac{1}{2}\mathrm{Re}[\Delta_P^e-\Delta_P^\mu]\mathrm{Re}[\Delta_P^e-3\Delta_P^\mu]+\frac{1}{2}\mathrm{Im}[\Delta_P^e-\Delta_P^\mu]\mathrm{Im}[\Delta_P^e+\Delta_P^\mu]+\mathcal{O}(\Delta^3)$$

at 95% C.L. Constraints on T-violating correlations in $\pi_{l2\gamma}$ decays, which result from Eq. (10), are model dependent. Any constraints on the coupling constants G_P^μ and G_P^e following from Eq. (10) are evaded, if there is a hierarchy in the coupling constants,

$$G_P^\mu/G_P^e=m_\mu/m_e. \quad (11)$$

Then the contributions to R cancel: indeed, if the μ-e hierarchy (11) is satisfied, then $\Delta_P^e=\Delta_P^\mu$ and the result for the ratio R coincides with the SM prediction. In this case any $\mathrm{Im}[\Delta_P]$ are allowed, and although the decays we discuss are rare processes $\mathrm{Br}(\pi\to e\bar{\nu}_e\gamma)=(1.61\pm 0.23)\times 10^{-7}$ [9,8], $\mathrm{Br}(\pi\to\mu\bar{\nu}_\mu\gamma)=(2.00\pm 0.25)\times 10^{-4}$ [18], even experiments with relatively low pion statistics have chances to observe T violation in $\pi_{l2\gamma}$ decays: the total number of charged pions should be $N_\pi\gtrsim 10^8$ for $\pi_{e2\gamma}$ and $N_\pi\gtrsim 10^5$ for $\pi_{\mu 2\gamma}$.

Note that to the leading order in Δ_P, the bound (10) constrains only the real parts of the coupling constants G_P^μ and G_P^e entering Eq. (1), while constraints on imaginary parts are weaker. Thus, for general G_P^μ and G_P^e [if the hierarchy (11) does not hold, i.e., if there is no cancellation between Δ_P^e and Δ_P^μ] one obtains $|\mathrm{Re}[\Delta_P]|\lesssim 10^{-3}$ and $|\mathrm{Im}[\Delta_P]|\lesssim 0.03$. Hence in this case experiments aimed at searching for T violation in $\pi_{l2\gamma}$ decays should have sufficiently large statistics: the total number of charged pions should be $N_\pi\gtrsim 10^{11}$ for $\pi_{e2\gamma}$ and $N_\pi\gtrsim 10^8$ for $\pi_{\mu 2\gamma}$. Note that in the $\pi_{\mu 2\gamma}$ case the contribution of the new interaction (1) to T-odd correlation is at best of the same order of magnitude as FSI effects. One can hope to discriminate between them only if they have different signs, i.e., $\mathrm{Im}[\Delta_P^\mu]$ is negative (cf. Figs. 1 and 3).

In models with $\mathrm{Re}[G_P]\sim\mathrm{Im}[G_P]$ and without the hierarchy (11), the bound (10) from $\pi\to l\nu$ decays implies

$|\mathrm{Im}[\Delta_P]| \lesssim 10^{-3}$, which significantly constrains the possible contribution of the new interaction (1) to T-odd correlation in $\pi_{l2\gamma}$ decays; namely, the contribution to the $\pi_{e2\gamma}$ decay should be of the same order as or weaker than that from the Standard Model FSI. The constraint on Δ_P^μ means that the effect is at least two orders of magnitude smaller than the FSI effects in $\pi_{\mu 2\gamma}$ decay. Nevertheless, as we discussed, in the case of $\pi_{e2\gamma}$ the difference in (x,y) distributions of FSI and four-fermion contributions may allow one to discriminate between the two if they are of the same order of magnitude, and even if the contribution of the four-fermion interaction (1) is somewhat weaker. On the other hand, an experiment aimed at probing T violation in $\pi_{e2\gamma}$ decay has to deal with very high pion statistics. Indeed, to test the four-fermion interaction (1) at the level allowed by $\pi \to e \nu_e$, i.e., at the level of 10^{-3}, one has to collect not less than 10^{13} charged pions, assuming statistical uncertainty only. For $\pi_{\mu 2\gamma}$ decay, even though the branching ratio is not very small, the (x,y) distributions of FSI and four-fermion contributions have similar shapes, while the expected effect is at least two orders of magnitude smaller than the FSI effects in $\pi_{\mu 2\gamma}$ decay, making it hardly possible to detect new sources of T violation in models without the hierarchy (11) and with $\mathrm{Re}[G_P^\mu] \sim \mathrm{Im}[G_P^\mu]$.

Overall, in the case of the hierarchy (11), decays $\pi \to l \nu$ do not constrain new T-violating interactions which can be searched for in relatively low statistics experiments, $N_\pi \gtrsim 10^8$ for $\pi_{e2\gamma}$ and $N_\pi \gtrsim 10^5$ for $\pi_{\mu 2\gamma}$. In the worst case of no hierarchy (11) and $\mathrm{Re} G_P \sim \mathrm{Im} G_P$, new T-violating interactions have little chance to be observed, and in $\pi_{e2\gamma}$ decay only.

III. A SIMPLE MODEL: HEAVY PSEUDOSCALAR EXCHANGE

To illustrate that the hierarchy (11) may appear naturally in low energy physics (thus the T-violating effects may be sufficiently large) we present below an example of a model which can lead to the effective interaction (1) with coupling constants obeying the hierarchy (11). As we show, this model may be considered as ‘‘proof by example’’ of the fact that in extensions of the Standard Model, the large T-violating low energy interaction (1) may arise without any contradiction to existing experiments.

Let us assume that in addition to the SM content there exists a heavy charged pseudoscalar field coupled to both lepton and quark sectors via the following Lagrangian:

$$\mathcal{L}_H = Y_{ij} H^* \overline{d}_i \gamma_5 u_j + Y_e H \overline{\nu}_e (1+\gamma_5) e + Y_\mu H \overline{\nu}_\mu (1+\gamma_5) \mu + \mathrm{H.c.} \tag{12}$$

The mass of the charged ‘‘extra Higgs’’ particle H is supposed to be of the order of $M_H \sim 1$ TeV. We assume for simplicity that there are no mixing couplings in the leptonic sector. The new Yukawa coupling constants in the leptonic sector are supposed to obey the hierarchy $Y_\mu / Y_e = m_\mu / m_e$ with accuracy of (1–0.1)% at the scale M_H. Then even for the model with $\mathrm{Re}[Y] \sim \mathrm{Im}[Y]$, the bound from $\pi \to l \nu$ decays (10) gives no constraints on the contribution of the new interaction (12) to T-odd correlations in $\pi_{l2\gamma}$ decays.

At energies below M_H this Lagrangian leads to the following four-fermion interaction:

$$\mathcal{L}_Y = (G_{Pij}^e \overline{d}_i \gamma_5 u_j \cdot \overline{\nu}_e (1+\gamma_5) e + G_{Pij}^\mu \overline{d}_i \gamma_5 u_j \cdot \overline{\nu}_\mu (1+\gamma_5) \mu + G_P^{\mu e} \overline{\nu}_e (1+\gamma_5) e \cdot \overline{\mu} (1-\gamma_5) \nu_\mu + \mathrm{H.c.}) - G_{Pijkl} \overline{u}_i \gamma_5 d_j \cdot \overline{d}_k \gamma_5 u_l + G_P^{ee} \overline{\nu}_e (1-\gamma_5) e \cdot \overline{e} (1+\gamma_5) \nu_e + G_P^{\mu\mu} \overline{\nu}_\mu (1-\gamma_5) \mu \cdot \overline{\mu} (1+\gamma_5) \nu_\mu, \tag{13}$$

where

$$G_{P\,ij}^e = \frac{Y_{ij} Y_e}{M^2}, \qquad G_{Pij}^\mu = \frac{Y_{ij} Y_\mu}{M^2}, \quad G_{Pijkl} = \frac{Y_{ji}^* Y_{kl}}{M^2},$$
$$G_P^{\mu e} = \frac{Y_\mu^* Y_e}{M^2}, \qquad G_P^{ee} = \frac{Y_e^* Y_e}{M^2}, \quad G_P^{\mu\mu} = \frac{Y_\mu^* Y_\mu}{M^2}. \tag{14}$$

The μ-e hierarchy is natural in this model, since the corresponding couplings $G_P^e \equiv G_{P11}^e$ and $G_P^\mu \equiv G_{P11}^\mu$ emerge as a result of a Yukawa (Higgs-like) interaction with a heavy charged scalar. Assuming the μ-e hierarchy we obtain

$$\mathrm{Im}[\Delta_P^e] = 500 \cdot \mathrm{Im}[Y_{ud} Y_e] \cdot \left(\frac{1 \ \ \mathrm{TeV}}{M} \right)^2 = 2.5 \cdot \mathrm{Im}[Y_{ud} Y_\mu] \cdot \left(\frac{1 \ \ \mathrm{TeV}}{M} \right)^2, \tag{15}$$

$$\mathrm{Im}[\Delta_P^\mu] = 500 \cdot \mathrm{Im}[Y_{ud} Y_\mu] \cdot \left(\frac{1 \ \ \mathrm{TeV}}{M} \right)^2. \tag{16}$$

Although $\mathrm{Im}[\Delta_P^\mu]$ is significantly larger than $\mathrm{Im}[\Delta_P^e]$, the resulting contributions to the T-odd correlations are of the same order. Indeed, the asymmetry P_T is determined by the product of $\mathrm{Im}[\Delta_P]$ and the distribution function $[\sigma_V(x,y) - \sigma_A(x,y)]$ [see Eq. (8)]. The typical values of this function for $\pi_{2e\gamma}$ and $\pi_{2\mu\gamma}$ decays (see Fig. 1) exhibit a hierarchy inverse to one appearing in Eqs. (15),(16). Thus one concludes that a large observable T-violating effect (of the order of 1–0.1) in both $\pi_{e2\gamma}$ and $\pi_{\mu 2\gamma}$ is possible if $\mathrm{Im}[Y_{ud}Y_\mu]\simeq 1$–0.1.

In the following subsections we discuss the experimental constraints on the interaction (12).

A. Muon decay

If the μ-e hierarchy (11) holds, then the interaction (12) contributes to $\mu\to e\overline{\nu_e}\nu_\mu$ decay. The Standard Model contribution to the squared modulus of the matrix element summed over spin states

$$\overline{|M_{\rm SM}|^2}=128G_F^2(pq_1)(kq_2)$$

has to be compared with the contribution from the interaction (13):

$$\overline{|M_P|^2}=64|G_P^{\mu e}|^2(pq_2)(kq_1),$$

and with the interference contribution

$$2M_{\rm SM}\mathrm{Re}[M_P]=128\mathrm{Re}[G_P^{\mu e}]\frac{G_F}{\sqrt{2}}m_\mu m_e(q_1q_2).$$

Here p, k, q_1, and q_2 are the momenta of μ, e, $\overline{\nu_e}$, and ν_μ, respectively. Integrating over the neutrino momenta one obtains for the differential muon decay width

$$d\Gamma=\frac{(G_F^2+|G_P^{\mu e}|^2/2)m_\mu^5}{96\pi^3}(3-2\varepsilon)\varepsilon^2d\varepsilon+\frac{\mathrm{Re}[G_P^{\mu e}]G_Fm_em_\mu^4}{8\sqrt{2}\pi^3}(1-\varepsilon)\varepsilon d\varepsilon, \tag{17}$$

where $\varepsilon=k^0/k^0_{\rm max}=2k^0/m_\mu$.

The experimental constraint on the coupling constant $G_P^{\mu e}$ entering this formula can be obtained from the data on the ρ_0 parameter [8,19],

$$\rho_0=\frac{M_W^2}{M_Z^2\hat{c}_Z^2\hat{\rho}(m_t,M_H)},$$

which is the measure of the neutral to charged current interaction strength [here $\hat{c}_Z$ is the cosine of the Cabibbo angle in the modified minimal subtraction ($\overline{\rm MS}$) scheme, and $\hat{\rho}$ absorbs the SM radiative corrections]. At one sigma level this parameter equals [8]

$$\rho_0=0.9998^{+0.0011}_{-0.0006}.$$

If we postulate that neutral current interactions are precisely the same as in the SM, then we allow a new physics contribution to the muon decay width of the order of

$$|\Delta\Gamma|\lesssim 2(1-\rho_0)\Gamma_{\rm SM}$$

(the factor 2 appears here because $\Gamma\sim G_F^2\sim M_W^{-4}$). Comparing this deviation with the term proportional to $|G_P^{\mu e}|^2$ in Eq. (17) we obtain

$$|G_P^{\mu e}|<0.07G_F. \tag{18}$$

In fact, this is a fairly weak constraint. For example, setting $Y_\mu=1$ and assuming the μ-e hierarchy, one obtains from Eq. (18)

$$M>Y_\mu^2\sqrt{\frac{m_e}{m_\mu}\frac{1}{0.07G_F}}\simeq 75\ \ \mathrm{GeV}.$$

Taking account of the second term in Eq. (17) leads to an even weaker constraint on $\mathrm{Re}[G_P^{\mu e}]$. So, constraints on the leptonic part of the Lagrangian (12) are very modest.

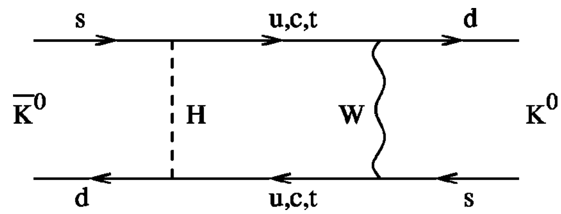

FIG. 4. Box diagram contributing to K^0-$\overline{K}^0$ mass difference.

B. K^0-$\overline{K}^0$ mixing

Let us estimate the contribution from the Lagrangian (12) to the mixing parameters in the neutral kaon system. These are the K_L-K_S mass difference Δm and the CP-violating parameter ϵ. They are expressed as follows:

$$\Delta m=\frac{2\mathrm{Re}\langle K^0|H_{\rm eff}|\overline{K}^0\rangle}{2m_K},\quad \epsilon=\frac{\mathrm{Im}\langle K^0|H_{\rm eff}|\overline{K}^0\rangle}{2\mathrm{Re}\langle K^0|H^{\rm full}_{\rm eff}|\overline{K}^0\rangle},$$

where $H_{\rm eff}$ is the effective four-fermion Hamiltonian, generated by the diagram of Fig. 4 and $H_{\rm eff}^{\rm full}$ is the effective Hamiltonian containing contributions both from the diagram presented in Fig. 4 and from the usual box diagram with two W bosons. We obtain

$$H_{\rm eff}=\frac{g^2}{8}\sum_{i,j=u,c,t}U_{is}U^*_{jd}Y_{js}Y^*_{id}\{[4\overline{d}_L\gamma^\mu s_L\cdot\overline{d}_L\gamma^\mu s_L]I_2(m_i,m_j)+[16\overline{d}_Rs_L\cdot\overline{d}_Ls_R-4\overline{d}_R\sigma^{\rho\mu}s_L\cdot\overline{d}_L\sigma^{\rho\mu}s_R]I_1(m_i,m_j)\} \tag{19}$$

where U is the Cabibbo-Kobayashi-Maskawa mixing matrix, $q_{R,L}=[(1\pm\gamma^5)/2]q$, and

$$g_{\rho\lambda}I_1(m_i,m_j)$$
$$=i\int\frac{d^4q}{(2\pi)^4}\frac{q_\rho q_\lambda}{(q^2-m_i^2)(q^2-m_j^2)(q^2-M_W^2)(q^2-M_H^2)}$$
$$=g_{\rho\lambda}\frac{1}{2(4\pi)^2M_H^2}A_1\left(\frac{m_i}{M_H},\frac{m_j}{M_H},\frac{M_W}{M_H}\right),$$

$$I_2(m_i,m_j)$$
$$=i\int\frac{d^4q}{(2\pi)^4}\frac{m_im_j}{(q^2-m_i^2)(q^2-m_j^2)(q^2-M_W^2)(q^2-M_H^2)}$$
$$=-\frac{m_im_j}{(4\pi)^2M_H^4}A_2\left(\frac{m_i}{M_H},\frac{m_j}{M_H},\frac{M_W}{M_H}\right),$$

with

$$A_k(\alpha,\beta,\gamma)=\int_0^1 dx\,dy\,dz\,dw\frac{\delta(x+y+z+w-1)}{[\alpha^2x+\beta^2y+\gamma^2z+w]^k},$$
$$k=1,2.$$

We calculate the matrix elements of the operators in Eq. (19) by making use of the "vacuum saturation" approximation, where nonvanishing matrix elements are

$$\langle K^0|\overline{d_L}\gamma^\mu s_L\cdot\overline{d_L}\gamma^\mu s_L|\overline{K^0}\rangle=\frac{2}{3}m_K^2f_K^2,$$
$$\langle K^0|\overline{d_R}s_L\cdot\overline{d_L}s_R|\overline{K^0}\rangle=\left[\frac{m_K^2}{12}+\frac{B_0^2}{2}\right]f_K^2.$$

Therefore

$$\langle K^0|H_{\rm eff}|\overline{K^0}\rangle=\frac{g^2f_K^2}{3(4\pi)^2}\sum_{i,j=u,c,t}U_{is}U_{jd}^*Y_{js}Y_{id}^*\times\left\{\frac{m_K^2+6B_0^2}{4M_H^2}A_1\left(\frac{m_i}{M_H},\frac{m_j}{M_H},\frac{M_W}{M_H}\right)-\frac{m_im_jm_K^2}{M_H^4}A_2\left(\frac{m_i}{M_H},\frac{m_j}{M_H},\frac{M_W}{M_H}\right)\right\}. \quad (20)$$

For $M_H\sim1$ TeV the second term in parentheses is negligible even for a t quark running in the loop. Thus we obtain

$$\Delta m=\frac{g^2f_K^2(m_K^2+6B_0^2)}{12(4\pi)^2M_H^2m_K}\sum_{i,j=u,c,t}\mathrm{Re}[U_{is}U_{jd}^*Y_{js}Y_{id}^*]\times A_1\left(\frac{m_i}{M_H},\frac{m_j}{M_H},\frac{M_W}{M_H}\right), \quad (21)$$

$$\epsilon=\frac{g^2f_K^2(m_K^2+6B_0^2)}{24(4\pi)^2M_H^2}\frac{1}{\mathrm{Re}\langle K^0|H_{\rm eff}^{\rm full}|\overline{K^0}\rangle}\times\sum_{i,j=u,c,t}\mathrm{Im}[U_{is}U_{jd}^*Y_{js}Y_{id}^*]\times A_1\left(\frac{m_i}{M_H},\frac{m_j}{M_H},\frac{M_W}{M_H}\right). \quad (22)$$

The contribution to the K^0-$\overline{K^0}$ mass difference 20 should be smaller than the experimental value $\Delta m=3.489\times10^{-12}$ MeV and the CP-violating parameter ϵ should not exceed the experimental value 2.271×10^{-3}, which are both consistently described by the CP-violating Cabibbo-Kobayashi-Maskawa (CKM) matrix in the Standard Model. This constrains the coupling constants Y_{ij}.

The strongest constraint comes from measurement of the CP-violating parameter ϵ, while, generally, the constraint from the K_L-K_S mass difference is an order of magnitude weaker. If one assumes that all coupling constants Y_{ij} are of the same order and have complex phases of order 1, this measurement requires them to be less than 10^{-4}, and if we set $Y_\mu=1$ then one obtains $P_T\sim10^{-4}$ [see Eqs. (15),(16) and Fig. 1]. Obviously, the situation changes for a nontrivial structure of the Y matrix. If the coupling with the first generation quarks is larger than with the second generation, then Y_{ud} (and, correspondingly, Δ_P) can be quite large, even if ϵ is kept small. As an example the values $|Y_{ud}=0.01|$, $|Y_{us}|=|Y_{cd}|=10^{-5}$, $|Y_{cs}|=10^{-4}$ are allowed. Another promising pattern is $Y_{ij}\propto U_{ij}$, i.e., the coupling constants Y are proportional to the CKM matrix. This is the case if the Y matrix is proportional to the unit matrix in the gauge basis of the quarks. The constraint disappears also in the case of aligned complex phases of Y_{ij}; in that case, the constraint from the kaon mass difference becomes important. And, of course, if Y_{is} is zero, then there is no contribution to ϵ and Δ_m at all.

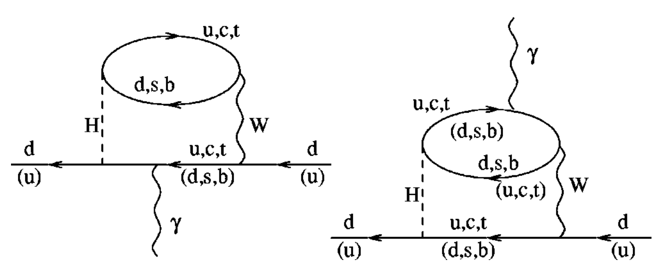

FIG. 5. Typical two-loop diagrams contributing to neutron dipole moment d_n.

Note that measurement of the parameter ϵ gives the strongest limit on our model in comparison with other CP-violating effects in kaon physics. The contribution to ϵ'/ϵ may be estimated following the lines of Ref. [20]. It is negligible for any parameters allowed by the requirement $\epsilon < \epsilon_{SM}$. Transverse muon polarization in $K_{\mu 2\gamma}$ decay is relevant only for the special choice of $Y \sim U$, since in this case the interaction (13) leads to P_T in this process of the same order as in $\pi_{e2\gamma}$. For other choices of Y the transverse muon polarization in $K_{\mu 2\gamma}$ is much smaller than in pion decays.

C. Neutron dipole moment

In our model, heavy charged scalar particles give a contribution to the neutron dipole moment at two-loop level (see Fig. 5). The contribution of the diagrams in Fig. 5 can be estimated as

$$\frac{d_n}{e} \sim \frac{(g/2\sqrt{2})^2}{16\pi^2} \sum_{i,j,k} \frac{Y_{dj}U_{jd}Y^*_{ik}U^*_{ki}}{16\pi^2} \frac{m_i f(m_i^2/M_H^2, m_k^2/M_H^2, m_j^2/M_H^2, M_W^2/M_H^2) + m_k g(m_i^2/M_H^2, m_k^2/M_H^2, m_j^2/M_H^2, M_W^2/M_H^2)}{M_H^2}, \tag{23}$$

where the dimensionless functions f and g are of order 1. For $m_i = 10$ MeV, $M_H \simeq 1$ TeV, and $Y_{ij} \simeq 1$ we have $d_n/e \sim 10^{-27}$ cm, which is two orders of magnitude smaller than the current limit. For the second generation of quarks running in the loop we have $m_i \lesssim 1$ GeV, which still allows corresponding Yukawa couplings to be of order 1. All special forms of the Y matrix described in the previous subsection lead to acceptable contributions to the neutron dipole moment. Thus, the constraints on the parameters of the Lagrangian (12) from the K-$\bar{K}$ mass difference are more stringent than current bounds from neutron dipole moment measurements.

To summarize the results of this section, the new interactions (12) cannot lead to significant T-violating effects in $\pi_{l2\gamma}$ decays if the Yukawa couplings do not exhibit a special pattern. In that case the most stringent constraints come from the measurement of the ϵ parameter. However, there are a number of special cases when these and other constraints do not apply, and T violation in $\pi_{l2\gamma}$ decays is allowed to be sufficiently large. Hence the lepton transverse polarization asymmetry in $\pi_{l2\gamma}$ is an interesting probe of the structure of new interactions at the TeV scale.

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